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Procedia Engineering 89 (2014) 59 – 68

Procedia Engineering

www.elsevier.com/locate/procedia

16th Conference on Water Distribution System Analysis, WDSA 2014

A Simulation-Optimization Approach for Reducing Background Leakage in Water Systems

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Abstract

This paper forms an entry to the battle of background leakage assessment for water networks (BBLAWN). The proposed methodology for this problem is a sequential assessment of intervention types. In an initial stage, a diagnosis of the network is performed through simulating its hydraulic behaviour with no infrastructure or operational modifications. An optimization technique is developed to recommended improvements of a particular type, such as pipes to replace. These techniques are applied sequentially to yield a list of suggested improvements for the network. Our approach requires a hydraulic model that simulates background leakage, custom implementations of heuristic algorithms and optimization solvers.

Peer-review under responsibility of the Organizing Committee of WDSA 2014

Keywords: water networks; background leakage; optimization

1. Introduction

Water loss from leaking pipes is a growing problem for aging infrastructure systems. As pipe networks within cities reach the end of their useful life, utilities face complex decisions about investment and operation for these systems. An especially pressing problem for older water networks is background leakage–a pressure dependent loss of water from the system that is always present. In an effort to assemble methods for controlling background leakage, the Water Distribution Systems Analysis Conference held in Bari, Italy, July 2014 is hosting the ”Battle of background leakage assessment for water networks (BBLAWN)’. This paper and related materials comprise an entry to the competition.

The BBLAWN problem is to propose a methodology for recommending changes to the design and operation of a water distribution system to minimize total expenditure while meeting service requirements. Operational costs include energy for pumping on a time of use tariff and a penalty for water lost to background leakage. Design options include replacing or paralleling existing pipes, installing pressure reducing valves, adding storage capacity, and adding pumps. A summary of the problem statement is given in Table 1; complete information is provided in [1].

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Peer-review under responsibility of the Organizing Committee of WDSA 2014
doi:10.1016/j.proeng.2014.11.160

The following sections describe a method to assess and control background leakage on water networks. The methodology is first described at a high level and then individual elements are treated in more detail. One notable contribution is a new simulation approach for water networks with background leakage based on fixed-point iteration. A simulator implementing this technique is combined with an optimizer to set tank levels and to choose pipes for replacement. A mixed-integer non-linear programming formulation of the PRV placement problem is given. Results obtained by applying the methodology on the challenge problem are described next. Through a combination of operational and design changes, the annual cost of the C-Town network is reduced from 3.9 to 1.45 million Euro.

Nomenclature

d_i	water demand at node i [L/s]
e_i	elevation at node i [m]
h_f	frictional head loss [m]
h_p	head added by a pump [m]
p_i	pressure at node i [m]
v_k	valve indicator for pipe k [$\in \{0, 1\}$]
Cd	Epanet emitter discharge coefficient
d_k^{leaks}	background leakage along pipe k
α_k	Leakage coefficient of pipe k [-]
β_k	Leakage coefficient of pipe k [$m^{2-\alpha/s}$]
D_k	Pipe diameter [m]
L_k, L	Pipe length [m], or the set of links
M	Big M constant
N_n	Number of nodes in the network
N_p	Number of pipes in the network
N_v	Number of valves in the network
P	set of pumps
Q	Flow rate [m^3/s]

2. Methods

2.1. Overall solution methodology

The solution methodology proposed here decomposes the overall problem into smaller more tractable problems aimed at a single type of decision. Examining each problem individually has the advantages of simplifying implementation of software and interpretation of results and allowing parts of the problem to be examined in parallel. The disadvantage of such a decomposition is that interactions between decisions may not be optimal. The proposed solution methodology proceeds through several steps:

1. Simulate the network using pump control levels based on engineering judgement
2. Select locations for new pressure reducing valves using mixed integer non-linear programming
3. Choose pipes to replace or re-size by simulation-optimization
4. Adjust pump control levels by simulation-optimization
5. Manually check pump replacements and tank additions
6. Re-run the level control optimization to ensure feasibility
7. Perform a final feasibility check

These steps examine design and operational changes to reduce the long-term cost of water distribution. In the existing system, background leakage comprises the majority of costs and so leakage reduction is prioritized. Pressure reducing valves, being inexpensive relative to pipe replacements are treated first. Next, the replacement and resizing of

Table 1: Problem summary

minimize	annual costs = energy + leakage + capital improvement
s.t.	minimum pressure of 20m at demand nodes, positive pressure for other nodes
	storage tanks maintain a level above zero and recover to their original level
where	energy cost varies over time and pumping efficiency follows a parabolic curve
	leakage is proportional to pressure to a power and leakage is valued at 2 Euro per cubic meter
	annual costs are provided for various capital improvements:
	replacing and paralleling pipes, adding pumps, adding hydraulic control valves, enlarging tanks
	pumps are controlled by tank level

individual pipes is considered. With cost effective measures in place for background leakage, reductions in electricity costs are pursued by changing pump controls, replacing pumps, and adding tanks. Temporal pattern for the setting of each PRV were not pursued as the overall effect is expected to be small. Another run of the level control optimization accounts for the new valve settings in the pump controls. A last feasibility check confirms that constraints are met.

2.2. Leakage simulation model

A hydraulic simulation of the water network equations, including background leakage, was essential to evaluate elements the objective function. The inclusion of pipe leakage in the mass balance equations rendered the usual simulation tools such as Epanet unsuitable, as Epanet does not support pressure dependent terms in the mass balance equations. The approach taken was to transform the leakage simulation problem into an equivalent formulation for which Epanet can be applied.

To simulate the leakage equations, a Picard iteration technique using the emitters feature in Epanet was developed. Emitters model pressure dependent demands at each node [2]. The idea behind the method is to calculate an emitter coefficient for each node and each timestep such that the emitter demand at the node matches the leakage demand allocated to the node by the background leakage model. In Epanet, emitters model pressure dependent demand at the node level using a pressure exponent and discharge coefficient [2]:

$$Q_{emitter} = CdP^{0.5}\,, \tag{1}$$

where Cd is a discharge coefficient. The mass balance on each node can be written to include the emitter term:

$$\sum_k Q_{j,i} = d_j + Cd_j P_j^{0.5}\,. \tag{2}$$

The problem treated here models the background leakage for pipe k connecting node i and node j as a function of the average pressure on the pipe, as

$$d_k^{\text{leaks}} = \beta_k L_k \left(\max\left\{\frac{P_i + P_j}{2}, 0\right\}\right)^{\alpha_k}\,, \tag{3}$$

Assuming nonnegative pressures, this means that, allocating half of the background leakage to each node, the mass balance equation for node j is

$$\sum_k Q_{i,j} = d_j + \sum_k \frac{1}{2}\beta_k L_k \left(\frac{P_i + P_j}{2}\right)^{\alpha_k}\,. \tag{4}$$

Subtracting Eq (4) from Eq (2) yields the emitter discharge coefficient in terms of the background leakage model:

$$Cd_j P_j^{0.5} = \sum_k \frac{1}{2}\beta_k L_k \left(\frac{P_i + P_j}{2}\right)^{\alpha_k}\,. \tag{5}$$

Eq (5) can be used to find the value of the emitter discharge coefficient that satisfies the background leakage model for each node and time step. A fixed-point technique is used to move from iteration level n to iteration level $n+1$

$$Cd_j^{n+1} = \frac{1}{(P_j^n)^{0.5}} \sum_k \frac{1}{2}\beta_k L_k \left(\frac{P_i^n + P_j^n}{2}\right)^{\alpha_k}\,. \tag{6}$$

The iteration stops when changes to Cd_j are sufficiently small. The technique described above was implemented by modifying the source code of Epanet.

2.3. Optimal Placement of Pressure Reducing Valves

Possible placements of pressure reducing valves to minimize leakage were explored using a mixed integer nonlinear programming approach. The model formulation used here takes the objective function and mass balance constraint of [3]: leakage is minimized and included in the mass balance equation. Energy conservation taking into account directionality of the pipe flow is modeled using a pair of constraints suggested by [4]. Additional network elements including PRVs, pumps, and check valves are modeled as described below.

A water network comprised of N_n nodes and N_p pipes, pumps, and valves, is modeled as a directed graph with N_n vertices and $2N_p$ edges called links. Nodes are numbered $i = 1...N_n$ and nodal quantities include demand d_i, elevation e_i, and pressure p_i. In contrast to the notation of the previous section, links are identified by the ordered pair of source and target node. Link quantities include flow rate $Q_{i,j}$, head loss $h_f(Q)_{i,j}$, leakage $QS_{i,j}$ and a binary indicator for the presence of PRVs $v_{i,j}$, pumps, and check valves. We denote the set of links by L, the links with pumps as $P \subset L$. The optimization problem is to minimize leakage by placing N_v pressure reducing valves on the network links and determining their set points. The optimization problem with the objective is to minimize system leakage is:

$$
\begin{aligned}
&\text{(VP-MINLP)}\ \min && \sum_k d_k^{\text{leaks}} && && \text{(7a)}\\
&\text{such that} && \sum_m Q_{m,i} - \sum_l Q_{i,l} = d_i + 0.5 \sum d_{i,j}^{\text{leaks}} && && \text{(7b)}\\
& && Q_{i,j}(p_i + e_i - p_j - e_j - h_f(Q)_{i,j}) \geq 0, && (i,j) \in L \setminus P && \text{(7c)}\\
& && p_i + e_i - p_j - e_j - h_f(Q)_{i,j} - Mv_{i,j} \leq 0, && (i,j) \in L \setminus P && \text{(7d)}\\
& && p_i + e_i - p_j - e_j + h_p(Q)_{i,j} = 0, && (i,j) \in P && \text{(7e)}\\
& && 0 \leq Q_{i,j} \leq Q_{\max} \quad \text{and} \quad p_{i,\min} \leq p_i \leq p_{\max} && && \text{(7f)}\\
& && v_{i,j} + v_{j,i} \leq 1 \quad \text{and} \sum_{(i,j)\in E} v_{i,j} \leq N_v && v_{i,j} \in \{0,1\} && \text{(7g)}
\end{aligned}
$$

Eq. (7b) represents mass conservation for the ith node. $Q_{m,i}$ are inbound flows and $Q_{i,l}$ the outbound flows for node i. As described in (3), half of the leakage for pipes connected to the node are allocated to the demand.

Energy conservation for the link i, j is written as two constraints, (7c) and (7d). For flow from node i to node j, Eq (7c) allows a head difference between nodes greater than or equal to friction loss along the pipe. To simplify the optimization a quadratic approximation of h_f was used following [5]. Eq (7d) is formulated as a big-M constraint, in order to model the switch between the case where there is no PRV on link i, j and the head difference equals the pipe friction, and the case where a PRV is placed on i, j and the head difference exceeds the pipe friction. Disabling the constraint is achieved by adding a large term $Mv_{i,j}$ which is non-zero for $v_{i,j} = 1$ only. Furthermore, a solution with $Q_{i,j} > 0$ and $Q_{j,i} > 0$ is infeasible [4]. Formulating energy conservation this way avoids additional binary variables to account for the flow direction or constraints with non-smooth functions in Q or p. In case a pump is located on a link i, j, the constraint (7e) needs to be satisfied instead of (7c)- (7d), in order to model the energy added by the pump. In our model the pump curve h_p is a quadratic function in Q.

In addition to constraints enforcing energy and mass conservation, box constraints are applied for the flow and pressure (7f). In this work, flows vary from zero through the highest flow used to fit the quadratic. Pressures vary from the lowest pressure allowed on the network to an appropriately large value determined by simulating the network. The lowest allowable pressure $p_{i,\min}$ is set to be 0 or 20 m, as set in the problem description. It limits the leak reduction attainable on the system

The remaining constraints (7g) reflect the use of a binary variable $v_{i,j}$ to indicate the presence of a PRV on a link. Only one PRV is allowed per pipe and the total number of PRVs may not exceed N_v.

2.4. Optimal Pipe Replacement

As a step to reduce background leakage and to minimize the cost of operation of the C-town network, pipe replacement and resizing is explored. The pipe replacement and resizing is performed through a heuristic inspired from the profit-to-cost approach that is typically used to find good solutions for the 0-1 knapsack problem [6]. As a first step, the heuristic orders the pipes by decreasing order of the profit-to-cost ratio. The profit for a particular pipe in this context is the total background leakage cost for this pipe while the cost is the cost of replacing this pipe by a newer one with identical diameter. Then starting with the pipe with the highest profit-to-cost ratio, the pipes are replaced. Since the exact impact of pipe replacement on the operation of the system cannot be known without a simulation, after each pipe replacement the new network is simulated and the pipe is replaced only if the new cost of the networks including the replacement cost is less than the currently best found cost. To reduce the cost further, pipe resizing is also considered where a replaced pipe can be downsized and a cheaper pipe is used; thus reducing the capital cost or a pipe might be upsized to reduce the operational cost of the network at the expense of additional capital cost of a larger pipe. The pipe size for each replaced pipe is decided by iterating among the potential diameters and computing through simulation the cost that results from each pipe size while ensuring the feasibility of the network. The steps of the pipe replacement and resizing algorithm are as follows:

1. Simulate the network and compute the leakage cost for each pipe and the total network cost.
2. Let *current best cost* be the simulated network cost.
3. Compute the profit-to-cost ratio for each pipe and sort the profit-to-cost ratios in decreasing order.
5. Starting with the pipe with the highest profit-to-cost ratio, REPEAT
   5.1 Replace the selected pipe by a newer one with identical diameter (choose nearest diameter if current is not available).
   5.2 Simulate the new network.
   5.3 If simulated network cost is less than *current best cost*.
      5.3.1 Iterate among all possible pipe diameters and simulate each instance.
      5.3.2 Choose the pipe diameter that leads to the least total cost.
      5.3.3 Update *current best cost*.
   5.4 Else, keep old pipe.

2.5. Optimal Level Switches

Tank level switches are used to control the operation of pumps. While pump operation directly translates to energy cost, regulating the levels in tanks and hence the pressure in the system affects the background leakage and related costs as well. Thus it is necessary to optimize the settings of the level switches taking into account both energy and leakage. In addition, the controls must be set while satisfying a number of operational constraints that include:

- The tanks must never be emptied.
- The final level in each tank should not be less than its starting level.
- The minimum pressure at the nodes that have demand should be greater than 20.

Although, the level switches can be set at any setting between 0 and the maximum level of the respective tank, it is reasonable to represent possible control levels at 0.1 m increments. However, exhaustively checking all possible settings combinations is impossible due to the large number of options thus we propose to optimize the settings of the level switches using a random walk heuristic. This approach to general meta-heuristics is a low overhead and partial search method that has attractive convergence properties [7]. A random walk explores a sequence of states $s_0, \dots, s_n$ where each state s_i is chosen uniformly at random from among all the states that can be reached from s_{i-1}. While there is a number of possible implementations of random walk, we implement a grid walk [7] mainly due to the discretization of the possible controls levels. At each iteration of the grid walk, the neighborhood of the current solution s_i is explored and a new solution is chosen at random from among the possible feasible solutions.

Through computational testing, we observed that enforcing feasibility early in the algorithm constrains its performance. Thus we enforce feasibility in stages, where given an infeasible starting point that is likely to violate all of the operational constraints, the grid walk is allowed to move to new infeasible points until "tanks never be emptied" constraint is satisfied, thus enforcing this constraint in all the steps that follow. Similarly, the "final level in each tank should not be less than its starting level" is enforced following a step that reaches a point that is feasible to that constraint. The minimum pressure constraint in the nodes is enforced in a similar approach.

3. Results and Discussion

The multi-step approach outlined in the methods section was applied to the C-Town network specified for BBLAWN. The contribution of leakage simulation, pressure reducing valves, pipe replacements, level controls and other improvements are discussed in the following paragraphs.

Step 1: (Simulation) The simulation technique described above was implemented by modified and extending the source code of Epanet and was applied on the C-Town network. Validation of the simulator occurred in two ways. An initial manual check of selected nodes and pipes verified that equations were satisfied. The flow and pressure solution obtained by fixed point iteration was also compared with the solution obtained from solving the leakage equations using a Newton-Raphson technique. Both cases showed that an accurate solution was obtained. For the time steps in the C-town system, between 5-10 fixed point iterations were needed per time step. Convergence problems for leakage equations have been reported in the literature [8] but were not observed for the network studied. Although fixed point iteration lacks the quadratic convergence properties of Newton-type methods, fixed-point worked well here because the symmetry of the underlying matrix was preserved. This symmetry allowed for inversion using existing sparse matrix algorithms implemented in Epanet.

Step 2 (Valve Placement): The optimization problem VP-MINLP formulated in Eqs (7) was modeled in AMPL [9] and solved using Branch and Bound and interior point techniques implemented in Bonmin [10] and Ipopt [11].

Note that this optimization problem does not include dynamics of the network, but uses the conservation equations as a static constraint. The hour of largest demands hour 166 was used to give the demand constraint, as this hour represents the largest stress on the network. In the optimization all pumps are assumed to be switched on.

The optimization problem was solved for each of the DMAs individually, as their respective hydraulics are largely independent. Unsurprisingly, the behavior of the optimization method performed differently for different DMAs. The optimization problem is a mixed-integer, nonconvex optimization problem and the constraint set has a complicated topology. If the number of valves N_v exceeded 5 this led to significant convergence and feasibility problems, even using warm starts. Instances with an upper bound N_v of 2 or 3 were easily solved. The approach taken was to iterate the optimization looking for a small number of additional valves while retaining the previously placed valves.

For DMA 1 this approach was repeated until the optimization found no further placements reducing leakage. DMA 2, which contributes substantially to leakage, was very amenable to the approach and the iteration was repeated until no significant further improvements were found. The leakage in the remaining DMAs is comparatively small, resulting in small potential savings. For DMA 3 and 4 only a few valves were placed. DMA 5 represented a critical instance for the optimization approach. The assumption that the pumps are on and the neglect of the hydraulic dynamics resulted in solutions placing valves right after the pump, essentially trying to reduce pumping energy. Simulations invalidated such valve placements. As the leakage in DMA 5 is again comparatively small, this matter was not pursued.

A satisfying result of the optimization approach is that in many cases, the valves are placed in locations that intuition guided by engineering experience would also have considered. In total, 22 valves were placed.

Step 3: (Pipe replacement) After adding pressure reducing valves, the pipe replacement algorithm identified pipes that would be economical for replacement. Replacement pipes are visualized as colored segments in Fig. 1. A total of 345 pipes in the network are recommended for replacement. Of these, 148 retain the same size, while 101 replacements have a smaller and 96 have a larger diameter than the original pipe. Amongst the pipes that became larger, 63 were below the smallest available pipe size prior to the replacement and had their diameter increased to the smallest available size. Adding parallel pipes in the system was mentioned as a possibility in the problem description but was not pursued as a solution approach, being more expensive than simply replacing existing pipes.

Visual inspection of Fig. 1 shows that some replacement pipes have different sizes than their neighbors. This arrangement would not necessarily be recommended from a practical perspective. It arises from the algorithm applied here, where the most profitable pipes are replaced first. Once a replacement is made, an evaluation of other potential diameters is not carried out unless that pipe becomes again the worst offender in the network.

Step 4: (Level switches) At several stages in the leakage assessment methodology, the pump controls are evaluated to provide tank level feasibility. Controlling pumps by tank level provides some robustness against uncertainty in demands. Conversely, level switches are not open to optimization as they are only indirectly connected to time-varying energy costs. It was observed that many sets of control levels provide essentially the same electricity costs. The more difficult problem was finding control levels so that tanks recover their initial level by the end of the simulation.

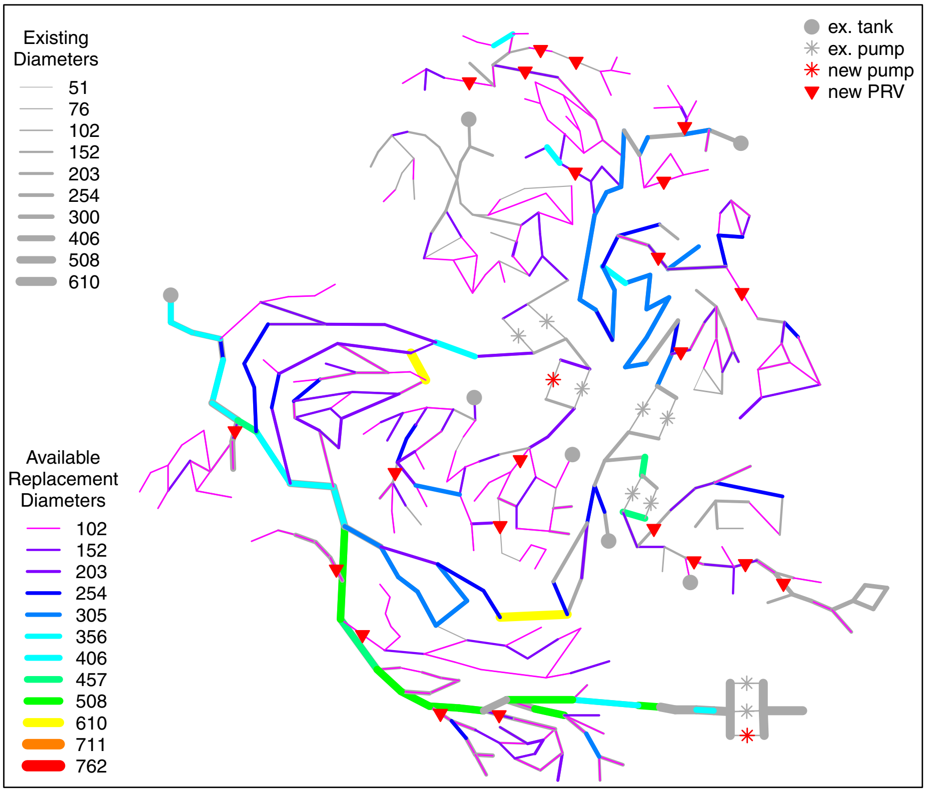

Fig. 1: C-Town network for BBLAWN showing existing and proposed elements.

Step 5: (Pump and tank replacements) Other potential improvements to the C-town system included tank expansions, pump upgrades, and timed settings for pressure reducing valves. Upgrades to pumping stations and storage tanks were evaluated by a trial and error. This analysis suggested upgrading pumps P1 and P7 to have higher efficiencies. Adding storage tank volume was found to increase leakage and electricity costs larger tanks took more time to drain and so maintained the system at higher pressures. Adjusting settings of PRVs by time was judged to have only marginal benefit compared to the implementation effort and was not pursued.

Steps 6 & 7: (Feasibility checks) A final tank level optimization was performed to ensure feasibility of the solution. The simulator developed in the course of this work showed full compliance with all constraints of the problem. The model parameters were then transformed into an instance which can be simulated using standard Epanet as reported in the supplementary files for this competition. The corresponding simulation confirmed the previous findings. The simulation using Epanet is very close to the one obtained using our simulator, as seen by the comparison of tank level results in Fig 1, where the black solid line shows the results of our simulator, while the red, dashed line shows the results of Epanet.

Collectively, the assessment methodology suggested infrastructural and operational changes to lower the total expenditure needed to run the network. In terms of capital improvements, suggested changes include 22 new pressure reducing valves, replacement of 345 pipes, and 2 new pumps. Location of new pipes, pumps and valves are shown in Fig. 1. The principle operational decisions include tank level switches for pump control and settings for pressure reducing valves. Switch levels and PRV settings are provided in the supplemental materials. Annual costs for the network based on the suggested changes appear in Table 2.

Table 2: Annual cost summary for proposed solution

Item	Cost (EUR)
Background Leakage	682,812
Pumping Energy	192,255
Pipe Replacements	559,584
Pump Upgrades	8,472
Tank Upgrades	0
Pressure Control Valves	10,662
Total	1,453,785

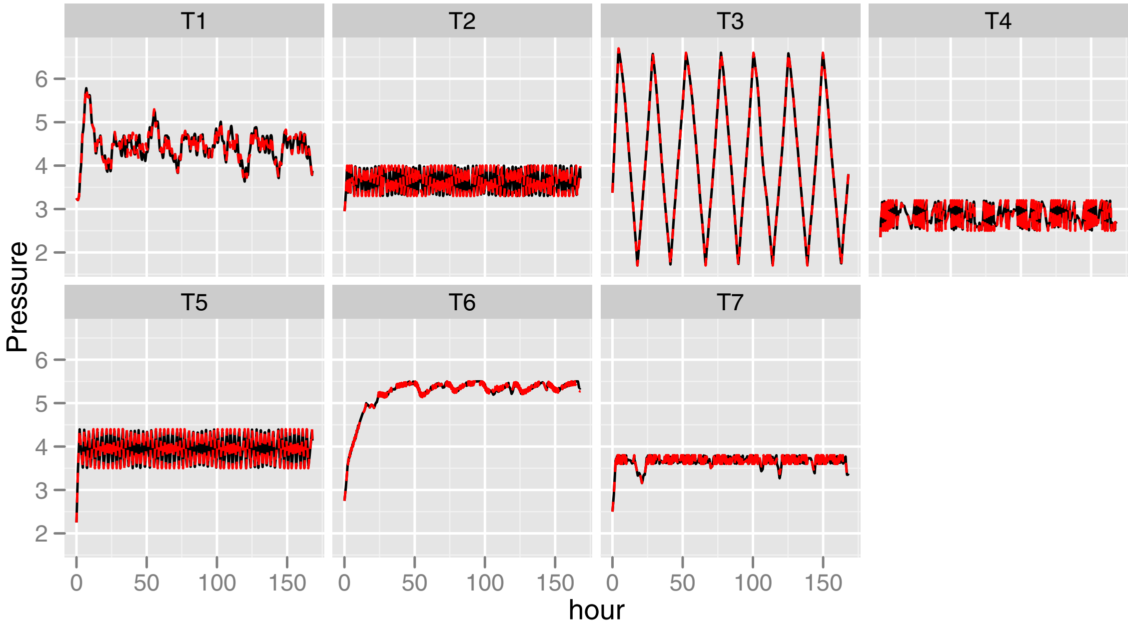

Fig. 2: Variation of tank levels over a one week simulation period with the recommended network and controls

With the suggested improvements, the system satisfies the pressure and tank constraints. Tanks do not empty during the simulation period and finish the simulation at or above their starting level (Fig. 2). The new system provides at least 20m of pressure at all demand nodes.

Fig. 3 shows the average pressure in the nodes and gives an impression of the impact of PRVs. The branches of the network that are regulated by these display quite favorable pressure behavior.

Even with the many new pressure control valves the maximum pressure on some demand nodes remains high with peak values of up to 100m.

4. Conclusions

The paper has described a method to assess and control background leakage on water networks. The method evaluates each class of improvement separately, employing a software tool to automate the analysis. This decomposition yields problems which are computationally tractable, allows solutions to proceed in parallel and adapts to other problems where fewer or different improvements are available. However, separate treatment may miss trade-offs between

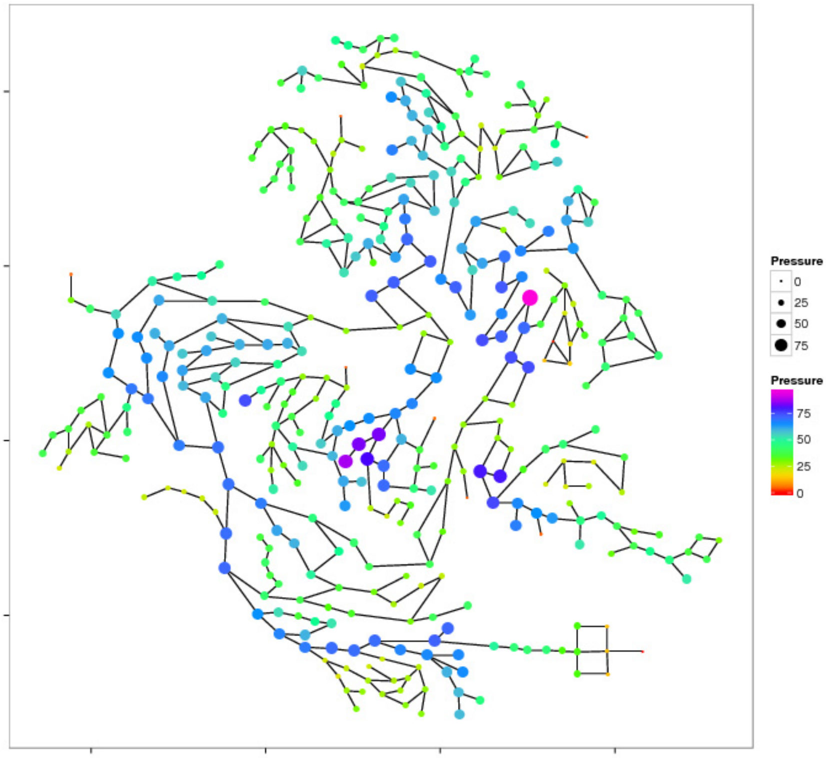

Fig. 3: Average pressure at demand nodes over the weekly simulation period

types of improvements such as pipe replacements and pressure reducing valves. Thus the order of evaluation is important. Evaluating the sample problem in a different order would probably yield a different solution. Such a heuristic approach provides no guarantee of solution quality or optimality.

The method was applied to the BBLAWN test problem which was posed specifically to examine background leakage. The resulting solution invests heavily in the network infrastructure to reduce leakage. A total of 22 PRVs are installed. Even after these additions, replacing 345 of the 432 pipes proved to be economical. These investments are justified by the high penalty imposed on leakage in this problem. In cases where a lower value is assigned to background leakage, other parts of the methodology such as pump control may have higher relative benefit.

The BBLAWN problem instance has thousands of parameters. The analysis reported here treats those parameters deterministically. Future work could consider robust optimization techniques that consider parameter uncertainty.

Acknowledgements

The work of Martin Mevissen on the AMPL version of the valve placement model is gratefully acknowledged.

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